Question	Scheme	Marks	AOs
1 (a)	Attempts $y.\frac{dx}{dt}=(2\sin 2t+3\sin t)\times 16\sin t\cos t$ and uses $\sin 2t=2\sin t\cos t$	M1	2.1
	Correct expanded integrand. **Usually** for one of $(R)=\int \underline{\underline{48\sin^2 t\cos t+16\sin^2 2t}}\,dt$; $(R)=\int \underline{\underline{48\sin^2 t\cos t+64\sin^2 t\cos^2 t}}\,dt$; $(R)=\int \underline{\underline{24\sin 2t\sin t+16\sin^2 2t}}\,dt$	A1	1.1b
	Attempts to use $\cos 4t=1-2\sin^2 2t=(1-8\sin^2 t\cos^2 t)$	M1	1.1b
	$R=\int_0^a 8-8\cos 4t+48\sin^2 t\cos t\,dt$ *	A1*	2.1
	Deduces $a=\frac{\pi}{4}$	B1	2.2a
		(5)	
(b)	$\int 8-8\cos 4t+48\sin^2 t\cos t\,dt=8t-2\sin 4t+16\sin^3 t$	M1 A1	2.1 1.1b
	$\left[8t-2\sin 4t+16\sin^3 t\right]_0^{\frac{\pi}{4}}=2\pi+4\sqrt{2}$	M1 A1	2.1 1.1b
		(4)	
		(9 marks)	
Notes:			

(a) Condone work in another variable, say $\theta \leftrightarrow t$ if used consistently for the first 3 marks

M1: For the key step in attempting $y.\frac{dx}{dt}=(2\sin 2t+3\sin t)\times 16\sin t\cos t$ with an attempt to use $\sin 2t=2\sin t\cos t$ Condone slips in finding $\frac{dx}{dt}$ but it must be of the form $k\sin t\cos t$

E.g. I $y.\frac{dx}{dt}=(2\sin 2t+3\sin t)\times k\sin t\cos t=(4\sin t\cos t+3\sin t)\times k\sin t\cos t$

E.g. II $y.\frac{dx}{dt}=(2\sin 2t+3\sin t)\times k\sin t\cos t=(2\sin 2t+3\sin t)\times\frac{k}{2}\sin 2t$

A1: A correct (expanded) integrand in t. Don't be concerned by the absence of $\int$ or dt or limits

$(R)=\int \underline{\underline{48\sin^2 t\cos t+16\sin^2 2t}}\,dt$ or $(R)=\int \underline{\underline{48\sin^2 t\cos t+64\sin^2 t\cos^2 t}}\,dt$

but watch for other correct versions such as $(R)=\int \underline{\underline{24\sin 2t\sin t+16\sin^2 2t}}\,dt$

M1: Attempts to use $\cos 4t = \pm 1 \pm 2\sin^2 2t$ to get the integrand in the correct form.

If they have the form $P\sin^2 2t$ it is acceptable to write $P\sin^2 2t = \frac{P}{2}(\pm 1 \pm \cos 4t)$

If they have the form $Q\sin^2 t\cos^2 t$ sight and use of $\sin 2t$ and/or $\cos 2t$ will usually be seen first.

There are many ways to do this, below is such an example

$$Q\sin^2 t\cos^2 t = Q\left(\frac{1-\cos 2t}{2}\right)\left(\frac{1+\cos 2t}{2}\right) = Q\left(\frac{1-\cos^2 2t}{4}\right) = Q\left(\frac{1}{4}-\frac{\cos^2 2t}{4}\right) = Q\left(\frac{1}{4}-\frac{1+\cos 4t}{8}\right)$$

Allow candidates to start with the given answer and work backwards using the same rules. So expect to see $\cos 4t = \pm 1 \pm 2\times\sin^2 2t$ or $\cos 4t = \pm 2\times\cos^2 2t \pm 1$ before double angle identities for $\sin 2t$ or $\cos 2t$ are used.

A1*: Proceeds to the given answer with correct working. The order of the terms is not important.
Ignore limits for this mark. The integration sign and the d*t* must be seen on their final answer.
If they have worked backwards there must be a concluding statement to the effect that they know that they have shown it. The integration sign and the d*t* must also be seen

E.g. Reaches $\int 48\sin^2 t\cos t + 64\sin^2 t\cos^2 t\,\mathrm{d}t$

Answer is

$$\int 8 - 8\cos 4t + 48\sin^2 t\cos t\,\mathrm{d}t$$
$$= \int 8 - 8\left(1 - 2\sin^2 2t\right) + 48\sin^2 t\cos t\,\mathrm{d}t$$
$$= \int 16\sin^2 2t + 48\sin^2 t\cos t\,\mathrm{d}t$$
$$= \int 64\sin^2 t\cos^2 t + 48\sin^2 t\cos t\,\mathrm{d}t$$

which is the same, ✓

B1: Deduces $a = \frac{\pi}{4}$. It may be awarded from the upper limit and can be awarded from (b)

(b)

M1: For the key process in using a correct approach to integrating the trigonometric terms.
May be done separately.
There may be lots of intermediate steps (e.g. let *u* = sin *t*) .
There are other more complicated methods so look carefully at what they are doing.

$\int 8 - 8\cos 4t + 48\sin^2 t\cos t\,\mathrm{d}t = ... \pm P\sin 4t \pm Q\sin^3 t$ where *P* and *Q* are constants

A1: $\int 8 - 8\cos 4t + 48\sin^2 t\cos t\,\mathrm{d}t = 8t - 2\sin 4t + 16\sin^3 t\,(+c)$

If they have written $16\sin^3 t$ as $16\sin t^3$ only award if further work implies a correct answer.
Similarly, 8*t* may be written as 8*x*. Award if further work implies 8*t*, e.g. substituting in their limits.
Do not penalise this sort of slip at all, these are intermediate answers.

M1: Uses the limits their *a* and 0 where $a = \frac{\pi}{6}, \frac{\pi}{4}$ or $\frac{\pi}{3}$ in an expression of the form $kt \pm P\sin 4t \pm Q\sin^3 t$ leading to an exact answer. Ignore evidence at lower limit as terms are 0

A1: CSO $2\pi + 4\sqrt{2}$ or exact **simplified** equivalent such as $2\pi + \frac{8}{\sqrt{2}}$ or $2\pi + \sqrt{32}$.

Be aware that $\int_0^{\frac{\pi}{4}} 8 - 8\cos 4t + 48\sin^2 t\cos t\,\mathrm{d}t = 8t \underset{=}{+} \lambda\sin 4t + 16\sin^3 t\,(+c)$ would lead to the correct answer but would only score M1 A0 M1 A0

Question	Scheme	Marks	AOs
2(a)	$...xe^x + ...e^x$	M1	1.1b
	$k\left(xe^x + e^x\right)$	A1	1.1b
	$\frac{d}{dx}\left(\sqrt{e^{3x}-2}\right) = \frac{1}{2}\times 3e^{3x}\left(e^{3x}-2\right)^{-\frac{1}{2}}$	B1	1.1b
	$\left(f'(x)=\right)\ \dfrac{(e^{3x}-2)^{\frac{1}{2}}("7"xe^x+"7"e^x)-"\frac{3}{2}"e^{3x}(e^{3x}-2)^{-\frac{1}{2}}\times"7"xe^x}{e^{3x}-2}$	dM1	2.1
	$f'(x)=\dfrac{7e^x\left(e^{3x}(2-x)-4x-4\right)}{2(e^{3x}-2)^{\frac{3}{2}}}$	A1	1.1b
		(5)	
(b)	$e^{3x}(2-x)-4x-4=0 \Rightarrow x(...e^{3x}\pm...)=...e^{3x}\pm...$	M1	1.1b
	$\Rightarrow x=\dfrac{2e^{3x}-4}{e^{3x}+4}$ *	A1*	2.1
		(2)	
(c)	Draws a vertical line $x=1$ up to the curve then across to the line $y=x$ then up to the curve finishing at the root (need to see a minimum of 2 vertical and horizontal lines tending to the root)	B1	2.1
		(1)	
(d)(i)	$x_2=\dfrac{2e^3-4}{e^3+4}=1.5017756...$	M1	1.1b
	$x_2=$ awrt 1.502	A1	1.1b
(ii)	$\beta=1.968$	dB1	2.2b
		(3)	
(e)	$h(x)=\dfrac{2e^{3x}-4}{e^{3x}+4}-x$ $h(0.4315)=-0.000297...$ $h(0.4325)=0.000947...$	M1	3.1a
	Both calculations correct and e.g. states: • There is a change of sign • e.g $f'(x)$ is continuous • $\alpha=0.432$ (to 3dp)	A1cao	2.4
		(2)	
		(13 marks)	

Notes

(a)

M1: Attempts the product rule on xe^x (or may be $7xe^x$) achieving an expression of the form $...xe^x \pm ...e^x$. If it is clear that the quotient rule has been applied instead which may be quoted then M0.

A1: $k\left(xe^x+e^x\right)$ (e.g. $7(xe^x+e^x)$) or equivalent which may be unsimplified (may be implied by further work)

B1: $\left(\frac{d}{dx}\left(\sqrt{e^{3x}-2}\right)=\right)\frac{1}{2}\times 3e^{3x}\left(e^{3x}-2\right)^{-\frac{1}{2}}$ (simplified or unsimplified)

dM1: Attempts to use the quotient rule. It is dependent on the previous method mark.
Score for achieving an expression of the form

$$(\mathrm{f}'(x) =) \ \frac{(\mathrm{e}^{3x}-2)^{\frac{1}{2}}(\text{"}7\text{"}x\mathrm{e}^{x}+\text{"}7\text{"}\mathrm{e}^{x}) - \text{"}\frac{3}{2}\text{"}\mathrm{e}^{3x}(\mathrm{e}^{3x}-2)^{-\frac{1}{2}} \times \text{"}7\text{"}x\mathrm{e}^{x}}{\mathrm{e}^{3x}-2}$$ or equivalent (do not be concerned by the constants for their "7" or their "$\frac{3}{2}$" which may be both 1)

If it is clear that the quotient rule has been applied the wrong way round then score M0.
Alternatively, applies the product rule. Score for achieving an expression of the form

$$(\mathrm{f}'(x) =) \ (\mathrm{e}^{3x}-2)^{-\frac{1}{2}}(\text{"}7\text{"}x\mathrm{e}^{x}+\text{"}7\text{"}\mathrm{e}^{x}) - \text{"}\frac{3}{2}\text{"}\mathrm{e}^{3x}(\mathrm{e}^{3x}-2)^{-\frac{3}{2}} \times \text{"}7\text{"}x\mathrm{e}^{x}$$ or equivalent (do not be concerned by the constants for their "7" or their "$\frac{3}{2}$" which may be both 1)

Do not condone invisible brackets.

A1: $(\mathrm{f}'(x) =) \ \dfrac{7\mathrm{e}^{x}\left(\mathrm{e}^{3x}(2-x)-4x-4\right)}{2(\mathrm{e}^{3x}-2)^{\frac{3}{2}}}$ following a fully correct differentiated expression.

You may need to check to see if (a) is continued after other parts for evidence of this.

Condone the lack of $\mathrm{f}'(x) =$ on the left hand side or allow the use of $\frac{\mathrm{d}y}{\mathrm{d}x}$ or y' instead.

Alternative (a) attempt using the triple product rule

e.g. $$\frac{\mathrm{d}}{\mathrm{d}x}\left(7x\mathrm{e}^{x}(\mathrm{e}^{3x}-2)^{-\frac{1}{2}}\right) = 7\mathrm{e}^{x}(\mathrm{e}^{3x}-2)^{-\frac{1}{2}} + 7x\mathrm{e}^{x}(\mathrm{e}^{3x}-2)^{-\frac{1}{2}} + 7x\mathrm{e}^{x} \times \left(-\frac{1}{2}\right) \times 3\mathrm{e}^{3x}(\mathrm{e}^{3x}-2)^{-\frac{3}{2}}$$

$$\Rightarrow \frac{\left(7\mathrm{e}^{x}+7x\mathrm{e}^{x}\right)(\mathrm{e}^{3x}-2) + 7x\mathrm{e}^{x} \times \left(-\frac{1}{2}\right) \times 3\mathrm{e}^{3x}}{(\mathrm{e}^{3x}-2)^{\frac{3}{2}}} = \frac{7\mathrm{e}^{x}\left(\mathrm{e}^{3x}-2+x\mathrm{e}^{3x}-2x-\frac{3}{2}x\mathrm{e}^{3x}\right)}{(\mathrm{e}^{3x}-2)^{\frac{3}{2}}} \Rightarrow \frac{7\mathrm{e}^{x}\left(\mathrm{e}^{3x}(2-x)-4x-4\right)}{2(\mathrm{e}^{3x}-2)^{\frac{3}{2}}}$$

M1: Attempts the product rule on $x\mathrm{e}^{x} \to \ldots x\mathrm{e}^{x} \pm \ldots \mathrm{e}^{x}$ which may be seen within the expression $\ldots \mathrm{e}^{x}(\mathrm{e}^{3x}-2)^{-\frac{1}{2}} \pm \ldots x\mathrm{e}^{x}(\mathrm{e}^{3x}-2)^{-\frac{1}{2}} + \ldots$ simplified or unsimplified.

A1: $k\left(x\mathrm{e}^{x}+\mathrm{e}^{x}\right)$ which may be seen within the expression $k\left(\mathrm{e}^{x}(\mathrm{e}^{3x}-2)^{-\frac{1}{2}} + x\mathrm{e}^{x}(\mathrm{e}^{3x}-2)^{-\frac{1}{2}}\right) + \ldots$ simplified or unsimplified.

B1: $\left(-\frac{1}{2}\right) \times 3\mathrm{e}^{3x}(\mathrm{e}^{3x}-2)^{-\frac{3}{2}}$ which may be seen within the expression $\ldots + k\left(x\mathrm{e}^{x} \times \left(-\frac{1}{2}\right) \times 3\mathrm{e}^{3x}(\mathrm{e}^{3x}-2)^{-\frac{3}{2}}\right)$ simplified or unsimplified.

dM1: A complete method using all three products (which may appear all on one line). Do not condone invisible brackets.

A1: As above in main scheme notes.

(b) **Note that if they do not have values $A = -4$, $B = -4$ in (a) (which may be seen later) then maximum score is M1A0***

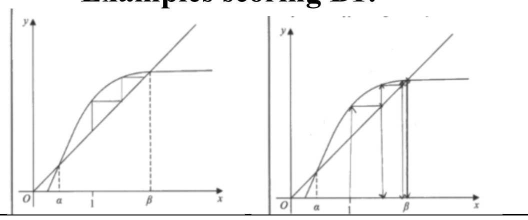
M1: Sets their $\mathrm{e}^{3x}(2-x)-4x-4$ equal to zero, collects terms in x on one side of the equation and non x terms on the other and attempts to factorise the side with x as a common factor. Condone sign slips only for this mark. Allow A and B to be used instead of "-4" and "-4"

A1*: Achieves the given answer with no errors including invisible brackets. If they do not reach the printed answer then it is A0. If they subsequently write $x_{n+1} = \dfrac{2e^{3x_n} - 4}{e^{3x_n} + 4}$ then isw

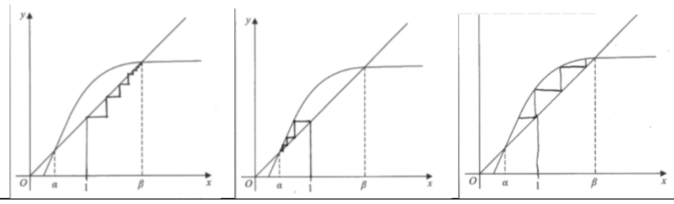
(c)

B1: Starting at $x_1 = 1$ look for at least 2 sets of vertical and horizontal lines drawn (may be dashes) tending to β. Condone a lack of arrows on the lines but the sequence of lines should finish at the point of intersection where the root is. Condone the initial vertical line not starting from the x-axis. Mark the intention to draw horizontal and vertical lines. If they have any lines to the left of $x = 1$ this is B0. If they use both diagrams and do not indicate which one they want marking, then the "copy of Diagram 1" should be marked.

Examples scoring B1:



Examples scoring B0:



(d)(i)

M1: Substitutes 1 into the iterative formula. The values embedded in the formula is sufficient for this mark. May be implied by awrt 1.50

A1: awrt 1.502 isw

(d)(ii)

dB1: 1.968 cao (which can only be scored if M1 is scored in (d)(i))

SC: If (d)(i) is rounded to 1.50 then allow 1.97 in (d)(ii) to score M1A0dB1 for (d)

(e)

M1: Attempts to substitute $x = 0.4315$ and 0.4325 into a suitable function and gets one value correct (rounded or truncated to 1sf). It is allowable to use a tighter interval that contains the root 0.4317388728…
If no function is stated then may be implied by their answers to e.g. $f'(0.4315)$, $f'(0.4325)$
You will need to check their calculation is correct.
Other possible functions include:

- $h(x) = x - \dfrac{2e^{3x} - 4}{e^{3x} + 4}$ (other way round to MS) $h(0.4315) = 0.0002974...$, $h(0.4325) = -0.0009479...$
- their $f'(x) = \pm\left(\dfrac{7e^{x}\left(e^{3x}(2-x) - 4x - 4\right)}{2(e^{3x} - 2)^{\frac{3}{2}}}\right)$

  (If correct A and B then $f'(0.4315) = \mp 0.005789...$, $f'(0.4325) = \pm 0.01831...$)
- their $g(x) = \pm\left(e^{3x}(2-x) - 4x - 4\right)$

  (If correct A and B then $g(0.4315) = \mp 0.002275...$, $g(0.4325) = \pm 0.007261...$)

A1: Requires

- Both calculations correct (rounded or truncated to 1sf)
- A statement that there is a change in sign and that their **function** is continuous (must refer to the function used for the substitution (which is not $f(x)$)

  Accept equivalent statements for $f'(0.4315) < 0$, $f'(0.4325) > 0$ e.g. $f'(0.4315) \times f'(0.4325) < 0$, "one negative one positive". A minimum is "change of sign and continuous" but do **not** allow this mark if the comment about continuity is clearly incorrect e.g. "because x is continuous" or "because the interval is continuous"
- A minimal conclusion e.g. "hence $\alpha = 0.432$", "so rounds to 0.432". Do not allow "hence root"

Question	Scheme	Marks	AOs
3(a)	$x^3 \to ...x^2$ and $3y^2 \to ...y\dfrac{dy}{dx}$	M1	1.1b
	$2xy \to 2y + 2x\dfrac{dy}{dx}$	B1	1.1b
	$3x^2 + 2x\dfrac{dy}{dx} + 2y + 6y\dfrac{dy}{dx} = ... \Rightarrow \dfrac{dy}{dx} = ...$	M1	2.1
	$\dfrac{dy}{dx} = -\dfrac{2y+3x^2}{2x+6y}$	A1	1.1b
		(4)	
(b)	$\dfrac{dy}{dx} = -\dfrac{2(5)+3(-2)^2}{2(-2)+6(5)}$ or e.g. $3(-2)^2 + 2(-2)\dfrac{dy}{dx} + 2\times5 + 6\times5\dfrac{dy}{dx} = 0 \Rightarrow \dfrac{dy}{dx} = ...$ $\left(-\dfrac{11}{13}\right)$	M1	1.1b
	$y - 5 = "\dfrac{13}{11}"(x+2)$	dM1	1.1b
	$13x - 11y + 81 = 0$	A1	2.2a
		(3)	
			(7 marks)

Notes

(a) Allow equivalent notation for the $\dfrac{dy}{dx}$ e.g. y'

M1: Attempts to differentiate $x^3 \to ...x^2$ **and** $3y^2 \to ...y\dfrac{dy}{dx}$ where … are constants

B1: Correct application of the product rule on $2xy$: $2xy \to 2x\dfrac{dy}{dx} + 2y$

Note that some candidates have a spurious $\dfrac{dy}{dx} = ...$ at the start (as their intention to differentiate) and this can be ignored for the first 2 marks

M1: For a valid attempt to make $\dfrac{dy}{dx}$ the subject, with exactly 2 different terms in $\dfrac{dy}{dx}$ coming from $3y^2$ and $2xy$. Look for $(...\pm...)\dfrac{dy}{dx} = ... \Rightarrow \dfrac{dy}{dx} = ...$ which may be implied by their working.

Condone slips provided the intention is clear.

For those candidates who had a spurious $\dfrac{dy}{dx} = ...$ at the start, they may incorporate this in their rearrangement in which case they will have 3 terms in $\dfrac{dy}{dx}$ and so score M0.

If they ignore it, then this mark is available for the condition as described above.

A1: $\dfrac{dy}{dx} = -\dfrac{2y+3x^2}{2x+6y}$ oe e.g. $\dfrac{dy}{dx} = \dfrac{-2y-3x^2}{2x+6y}, \dfrac{2y+3x^2}{-2x-6y}$ Isw once a correct expression is seen.

Note that it is sometimes unclear if the minus sign(s) is/are correctly placed and you may have to use your judgement. Evidence may be available in part (b) to help you decide if they have the correct expression.

(b)

M1: Substitutes x = -2 and y = 5 into $\frac{dy}{dx} = "-\frac{2y+3x^2}{2x+6y}"$

They must have x's and y's in their $\frac{dy}{dx}$ but condone slips in substitution provided the intention is clear.

As a minimum look for at least one x and at least one y substituted correctly.

Note that this mark may be implied by their value for $\frac{dy}{dx}$ and may be implied if, for example, they find the negative reciprocal or the reciprocal of $"-\frac{2y+3x^2}{2x+6y}"$ and then substitute x = -2 and y = 5

Alternatively, substitutes x = -2 and y = 5 into their attempt to differentiate and then rearranges to find a value or numerical expression for $\frac{dy}{dx}$

dM1: Attempts to find the equation of the normal using their gradient of the tangent and x = -2 and y = 5 correctly placed. Score for an expression of the form $(y-5) = "\frac{13}{11}"(x+2)$ or if they use $y = mx + c$ they must proceed as far as c =… Must be using the **negative reciprocal** of the tangent gradient.

Note that $y - 5 = \frac{2x+6y}{2y+3x^2}(x+2)$ is not a correct method unless the gradient is evaluated first *before* expanding.

A1: $13x - 11y + 81 = 0$ or any integer multiple of this equation including the "= 0", not just a, b, c given.
e.g., $26x - 22y + 162 = 0$ is likely if they don't cancel down their gradient.